Title: An Introduction to Functions by Calculating, Observing Growth Rates, and Interpreting the Graphs

Link To Outcomes:

- **Connections** Students will demonstrate the ability to connect mathematical topics within the discipline and business.
- **Statistics** Students will express data as graphs.
- **Habits of the Mind** Students will be open to new ideas about functions.
- **Reasoning** Students will look for patterns by interpreting graphs.
- **Communication** Students will express mathematical ideas orally and in writing.
- **Technology** Students will use a computer with a spreadsheet program.

Brief Overview:

This is a teacher-taught lesson in which the student will complete worksheets by using a spreadsheet on a computer. The students will look for patterns in various functions as they plot the data. The emphasis on this lesson will be an introduction to functions and their growth rates. The content is centered around the business concept of how a student will be paid on a monthly basis.

Grade/Level:

10-12, Advanced Level Mathematics and Business

Duration/Length:

The core lesson can be completed in two 40 minute class periods, with assigned homework. This may be extended to additional class periods.

Prerequisite Knowledge:

Students should have observational skills, basic mathematical skills, basic computer skills and spreadsheet skills. Teachers should have knowledge of a computer and a spreadsheet program.

Objectives:

- Learn to negotiate with another person.
- Hypothesize which payment plan gives the greatest gain; least gain.
- Express the growth rate of functions by using a spreadsheet.
- Interpret graphs.
- Express conclusions in writing.

Materials/Resources/Printed Materials:

- Computer
- Spreadsheet program
- Index cards
- Lab Activity Worksheets

Development/Procedures:

Divide the students into pairs. Give the worksheets and one index card to each student. Each student should put his/her name on the worksheets and index card.

Explain to the students the following scenario:

"You have just been hired by a major company to work on a special project for three years (36 months) and you must negotiate with your new boss for your salary. Both you and your boss must come to a consensus on how you are to be paid. As an employee, your goal is to obtain the most money that you can after a three year period. As a boss, your goal is to pay the least amount of money that you can to your employee."

Explain to the students the payment plans; the teacher may add or modify plans as appropriate for the students.

As each plan is explained, the teacher could have the students complete the first three rows of the worksheet. This will help to insure that the students understand each payment plan. The teacher may want to provide this information verbally or provide it as a handout for each student.

After all plans have been explained, but before the students complete the worksheets, each student should write down on their index card the payment plan that they would like to receive if they were going to be an employee.

Each student pair will then play rolls as boss and employee. One student will be the employee and the other the boss. The employee wants to get the highest possible pay, while the boss wants to pay the least amount. The employee will negotiate with the boss and they must agree on a payment plan. If there is an agreement, the boss should sign the employee's index card. Give the students exactly three minutes to negotiate. The teacher may serve as the final arbiter if the two students are at an impasse. The students should then reverse rolls and repeat this exercise. The teacher should then collect the index cards at this point.

The students should then enter the plans into a spreadsheet. This can be done one entry at a time. If the students are sophisticated computer users, the teacher may want to explore the possibility of using spreadsheet formulae to generate the plans. Are there any patterns that are developing?

The student will not finish this exercise in a forty-minute class period. The students should save their work. The students can be assigned some of the plans for homework as a paper and pencil exercise.
On the second day, the students should complete their spreadsheets.

Students should then be asked to graph each of the spreadsheet columns. This can be done for both the monthly payment plans and the running totals.

The students may want to explore further by changing the initial conditions for each plan. For example, the students may want to start each payment plan with $2000.00 for the first month.

Students should then write a summary of their observations. They should include in the summary an evaluation of their choices as both the employee and the boss.

- Were they influenced by the initial values?
- Were they influenced by the amount of raise that is given each month?
- What if the employment period was extend to five years, would this change your opinion of what function is the best (highest paying) or worst (least paying)?

Evaluation:

Students will be evaluated on the completion of their worksheets, graphs produced, and the written summary of their observations.

Extension/Follow Up:

- Have students design their own payment plans.
- How do the students handle rounding?
- How do the students handle scientific notation?
- Connection with sequences and series.
- Connection with limits.

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EXPLANATION OF PAYMENT PLANS

Plan A: You are paid $2000.00 the first month, with a one hundred dollar raise at the end of each month.

	Plan A	Total A
1	2000.00	2000.00
2	2100.00	4100.00
3	2200.00	6300.00
4	…	…

Plan B: You are paid $3000.00 the first month, with a fifty dollar raise at the end of each month.

	Plan B	Total B
1	3000.00	3000.00
2	3050.00	6050.00
3	3100.00	9150.00
4	…	…

Plan C: You are paid $4000.00 the first month, with a twenty-five dollar decrease in pay at the end of each month.

	Plan C	Total C
1	4000.00	4000.00
2	3975.00	7975.00
3	3950.00	11925.00
4	…	…

Plan D: You are paid $1.00 the first month, $8.00 the second month, $27.00 the third month, etc. Each month you receive the cube of the month number in dollars.

	Plan D	Total D
1	1.00	1.00
2	8.00	9.00
3	27.00	36.00
4	…	…

Plan E: You are paid $0.01 (one cent) the first month, $0.02 (two cents) the second month, $0.04 the third month, etc. Each month your pay is doubled.

	Plan E	Total E
1	0.01	0.01
2	0.02	0.03
3	0.04	0.04
4	…	…

Plan F: You are paid $80,000.00 on the first month, $40,000 on the second month, $20,000.00 on the third month, etc. Each month your pay is cut in half.

	Plan F	Total F
1	80000.00	80000.00
2	40000.00	120000.00
3	20000.00	140000.00
4	…	…

Plan G: You are paid $2,000.00 on the first month. Now find the square root of 2,000.00 to the nearest penny to calculate your raise. Your pay raise each month will be the square root of your previous month's pay.

	Plan G	Total G
1	2000.00	2000.00
2	2044.72	4044.72
3	2089.94	6134.66
4	…	…

Plan H: You are paid $2,000.00 on the first month. Now find 10% (to the nearest penny) of $2,000.00 to calculate your pay raise. Your pay raise each month will be 10% of your previous month.

	Plan H	Total H
1	2000.00	2000.00
2	2200.00	4200.00
3	2422.00	6622.00
4	…	…

Plan I: Your initial pay is 2000.00 and you get a pay increase the first month and a pay decrease the second month. A pay increase the third month and a pay decrease the fourth month. Every odd-numbered month you get a pay increase and every even-numbered month you get a pay decrease. The amount of increase or decrease will be a random three digit number. Each student will be generating a different set of numbers.

	Plan I	Total I
1	2000.00	2000.00
2	2735.00	4735.00
3	1943.00	6678.00
4	…	…

LAB ACTIVITY WORKSHEETS

Mo	Plan A	Total A	Plan B	Total B	Plan C	Total C
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LAB ACTIVITY WORKSHEETS

Mo	Plan D	Total D	Plan E	Total E	Plan F	Total F
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LAB ACTIVITY WORKSHEETS

Mo	Plan G	Total G	Plan H	Total H	Plan I	Total I
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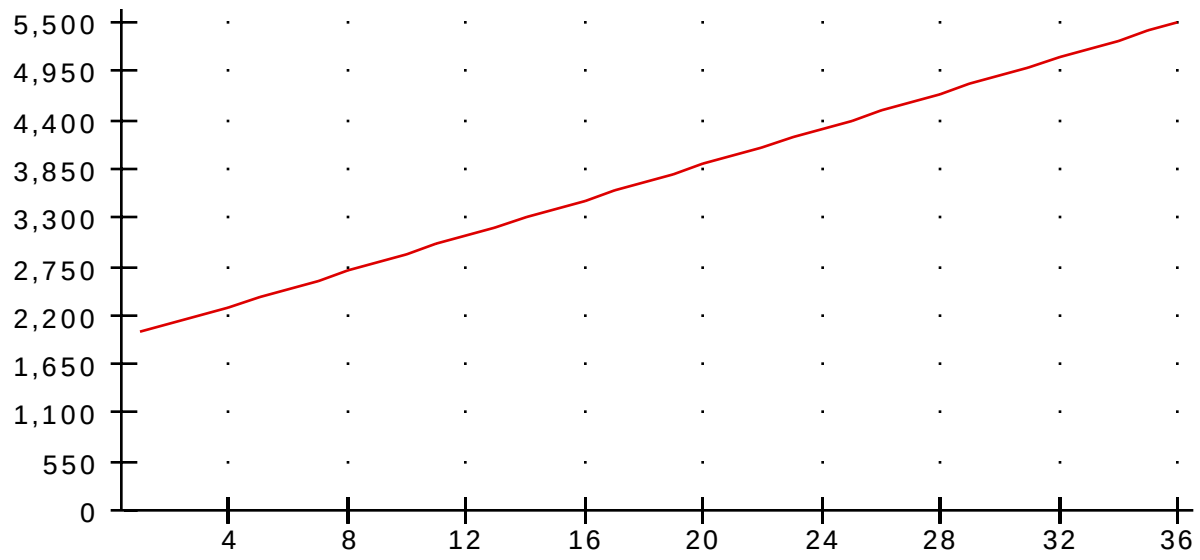
SOLUTIONS TO LAB ACTIVITY WORKSHEETS

Mo	Plan A	Total A	Plan B	Total B	Plan C	Total C
1	2,000.00	2,000.00	3,000.00	3,000.00	4,000.00	4,000.00
2	2,100.00	4,100.00	3,050.00	6,050.00	3,975.00	7,975.00
3	2,200.00	6,300.00	3,100.00	9,150.00	3,950.00	11,925.00
4	2,300.00	8,600.00	3,150.00	12,300.00	3,925.00	15,850.00
5	2,400.00	11,000.00	3,200.00	15,500.00	3,900.00	19,750.00
6	2,500.00	13,500.00	3,250.00	18,750.00	3,875.00	23,625.00
7	2,600.00	16,100.00	3,300.00	22,050.00	3,850.00	27,475.00
8	2,700.00	18,800.00	3,350.00	25,400.00	3,825.00	31,300.00
9	2,800.00	21,600.00	3,400.00	28,800.00	3,800.00	35,100.00
10	2,900.00	24,500.00	3,450.00	32,250.00	3,775.00	38,875.00
11	3,000.00	27,500.00	3,500.00	35,750.00	3,750.00	42,625.00
12	3,100.00	30,600.00	3,550.00	39,300.00	3,725.00	46,350.00
13	3,200.00	33,800.00	3,600.00	42,900.00	3,700.00	50,050.00
14	3,300.00	37,100.00	3,650.00	46,550.00	3,675.00	53,725.00
15	3,400.00	40,500.00	3,700.00	50,250.00	3,650.00	57,375.00
16	3,500.00	44,000.00	3,750.00	54,000.00	3,625.00	61,000.00
17	3,600.00	47,600.00	3,800.00	57,800.00	3,600.00	64,600.00
18	3,700.00	51,300.00	3,850.00	61,650.00	3,575.00	68,175.00
19	3,800.00	55,100.00	3,900.00	65,550.00	3,550.00	71,725.00
20	3,900.00	59,000.00	3,950.00	69,500.00	3,525.00	75,250.00
21	4,000.00	63,000.00	4,000.00	73,500.00	3,500.00	78,750.00
22	4,100.00	67,100.00	4,050.00	77,550.00	3,475.00	82,225.00
23	4,200.00	71,300.00	4,100.00	81,650.00	3,450.00	85,675.00
24	4,300.00	75,600.00	4,150.00	85,800.00	3,425.00	89,100.00
25	4,400.00	80,000.00	4,200.00	90,000.00	3,400.00	92,500.00
26	4,500.00	84,500.00	4,250.00	94,250.00	3,375.00	95,875.00
27	4,600.00	89,100.00	4,300.00	98,550.00	3,350.00	99,225.00
28	4,700.00	93,800.00	4,350.00	102,900.00	3,325.00	102,550.00
29	4,800.00	98,600.00	4,400.00	107,300.00	3,300.00	105,850.00
30	4,900.00	103,500.00	4,450.00	111,750.00	3,275.00	109,125.00
31	5,000.00	108,500.00	4,500.00	116,250.00	3,250.00	112,375.00
32	5,100.00	113,600.00	4,550.00	120,800.00	3,225.00	115,600.00
33	5,200.00	118,800.00	4,600.00	125,400.00	3,200.00	118,800.00
34	5,300.00	124,100.00	4,650.00	130,050.00	3,175.00	121,975.00
35	5,400.00	129,500.00	4,700.00	134,750.00	3,150.00	125,125.00
36	5,500.00	135,000.00	4,750.00	139,500.00	3,125.00	128,250.00

Mo	Plan D	Total D	Plan E	Total E	Plan F	Total F
1	1	1	0.01	0.01	80,000.00	80,000.00
2	8	9.00	0.02	0.03	40,000.00	120,000.00
3	27	36.00	0.04	0.07	20,000.00	140,000.00
4	64	100.00	0.08	0.15	10,000.00	150,000.00
5	125	225.00	0.16	0.31	5,000.00	155,000.00
6	216	441.00	0.32	0.63	2,500.00	157,500.00
7	343	784.00	0.64	1.27	1,250.00	158,750.00
8	512	1,296.00	1.28	2.55	625.00	159,375.00
9	729	2,025.00	2.56	5.11	312.50	159,687.50
10	1000	3,025.00	5.12	10.23	156.25	159,843.75
11	1331	4,356.00	10.24	20.47	78.12	159,921.88
12	1728	6,084.00	20.48	40.95	39.06	159,960.94
13	2197	8,281.00	40.96	81.91	19.53	159,980.47
14	2744	11,025.00	81.92	163.83	9.77	159,990.23
15	3375	14,400.00	163.84	327.67	4.88	159,995.12
16	4096	18,496.00	327.68	655.35	2.44	159,997.56
17	4913	23,409.00	655.36	1,310.71	1.22	159,998.78
18	5832	29,241.00	1,310.72	2,621.43	0.61	159,999.39
19	6859	36,100.00	2,621.44	5,242.87	0.31	159,999.69
20	8000	44,100.00	5,242.88	10,485.75	0.15	159,999.85
21	9261	53,361.00	10,485.76	20,971.51	0.08	159,999.92
22	10648	64,009.00	20,971.52	41,943.03	0.04	159,999.96
23	12167	76,176.00	41,943.04	83,886.07	0.02	159,999.98
24	13824	90,000.00	83,886.08	167,772.15	0.01	159,999.99
25	15625	105,625.00	167,772.16	335,544.31	0.00	160,000.00
26	17576	123,201.00	335,544.32	671,088.63	0.00	160,000.00
27	19683	142,884.00	671,088.64	1,342,177.27	0.00	160,000.00
28	21952	164,836.00	1,342,177.28	2,684,354.55	0.00	160,000.00
29	24389	189,225.00	2,684,354.56	5,368,709.11	0.00	160,000.00
30	27000	216,225.00	5,368,709.12	10,737,418.23	0.00	160,000.00
31	29791	246,016.00	10,737,418.24	21,474,836.47	0.00	160,000.00
32	32768	278,784.00	21,474,836.48	42,949,672.95	0.00	160,000.00
33	35937	314,721.00	42,949,672.96	85,899,345.91	0.00	160,000.00
34	39304	354,025.00	85,899,345.92	171,798,691.83	0.00	160,000.00
35	42875	396,900.00	171,798,691.84	343,597,383.67	0.00	160,000.00
36	46656	443,556.00	343,597,383.68	687,194,767.35	0.00	160,000.00

Mo	Plan G	Total G	Plan H	Total H	Plan I	Total I
1	2,000.00	2,000.00	2,000.00	2,000.00	2,000.00	2,000.00
2	2,044.72	4,044.72	2,200.00	4,200.00	2,795.00	4,795.00
3	2,089.94	6,134.66	2,420.00	6,620.00	2,458.00	7,253.00
4	2,135.66	8,270.32	2,662.00	9,282.00	2,459.00	9,712.00
5	2,181.87	10,452.19	2,928.20	12,210.20	1,544.00	11,256.00
6	2,228.58	12,680.77	3,221.02	15,431.22	1,650.00	12,906.00
7	2,275.79	14,956.55	3,543.12	18,974.34	1,235.00	14,141.00
8	2,323.49	17,280.05	3,897.43	22,871.78	1,267.00	15,408.00
9	2,371.70	19,651.74	4,287.18	27,158.95	442.00	15,850.00
10	2,420.40	22,072.14	4,715.90	31,874.85	471.00	16,321.00
11	2,469.59	24,541.73	5,187.48	37,062.33	-12.00	16,309.00
12	2,519.29	27,061.02	5,706.23	42,768.57	905.00	17,214.00
13	2,569.48	29,630.50	6,276.86	49,045.42	426.00	17,640.00
14	2,620.17	32,250.67	6,904.54	55,949.97	1,032.00	18,672.00
15	2,671.36	34,922.02	7,595.00	63,544.96	675.00	19,347.00
16	2,723.04	37,645.07	8,354.50	71,899.46	1,499.00	20,846.00
17	2,775.23	40,420.29	9,189.95	81,089.41	924.00	21,770.00
18	2,827.91	43,248.20	10,108.94	91,198.35	1,540.00	23,310.00
19	2,881.08	46,129.28	11,119.83	102,318.18	873.00	24,183.00
20	2,934.76	49,064.04	12,231.82	114,550.00	1,570.00	25,753.00
21	2,988.93	52,052.98	13,455.00	128,005.00	1,183.00	26,936.00
22	3,043.60	55,096.58	14,800.50	142,805.50	1,598.00	28,534.00
23	3,098.77	58,195.35	16,280.55	159,086.05	728.00	29,262.00
24	3,154.44	61,349.79	17,908.60	176,994.65	1,463.00	30,725.00
25	3,210.60	64,560.40	19,699.47	196,694.12	1,351.00	32,076.00
26	3,267.27	67,827.67	21,669.41	218,363.53	2,082.00	34,158.00
27	3,324.43	71,152.09	23,836.35	242,199.88	1,236.00	35,394.00
28	3,382.08	74,534.18	26,219.99	268,419.87	1,610.00	37,004.00
29	3,440.24	77,974.42	28,841.99	297,261.86	1,596.00	38,600.00
30	3,498.89	81,473.31	31,726.19	328,988.05	2,494.00	41,094.00
31	3,558.05	85,031.36	34,898.80	363,886.85	1,914.00	43,008.00
32	3,617.69	88,649.05	38,388.68	402,275.53	1,972.00	44,980.00
33	3,677.84	92,326.89	42,227.55	444,503.09	1,157.00	46,137.00
34	3,738.49	96,065.38	46,450.31	490,953.40	1,463.00	47,600.00
35	3,799.63	99,865.01	51,095.34	542,048.74	940.00	48,540.00
36	3,861.27	103,726.28	56,204.87	598,253.61	1,479.00	50,019.00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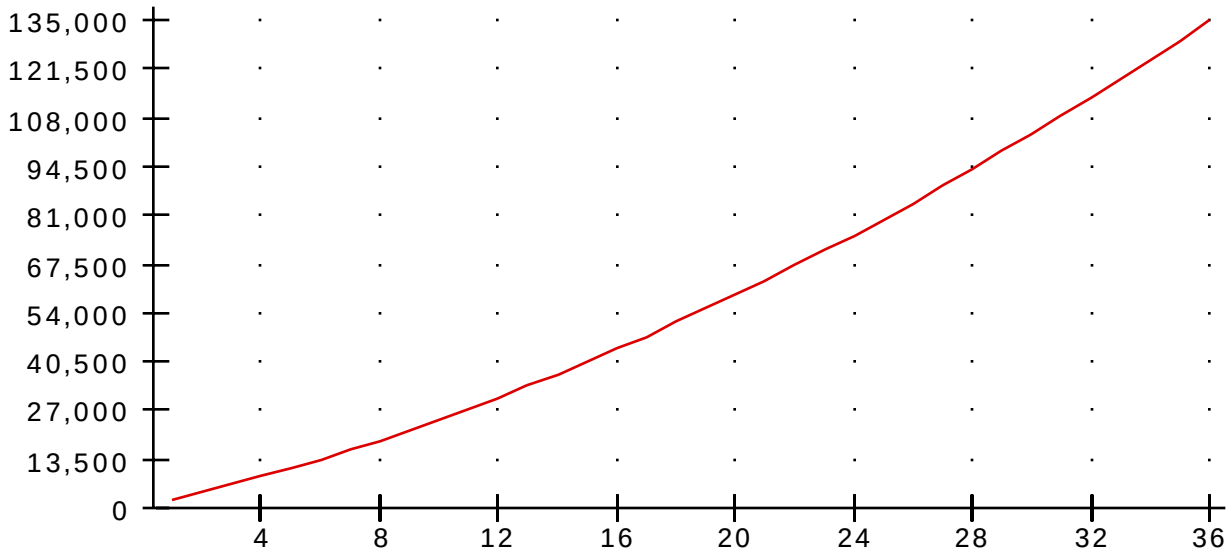
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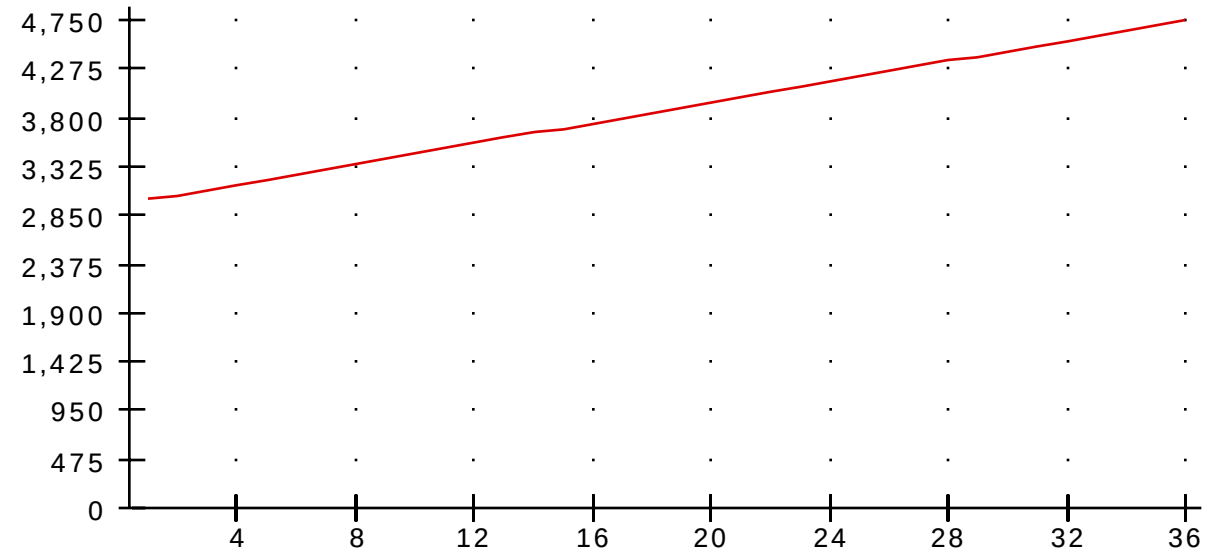
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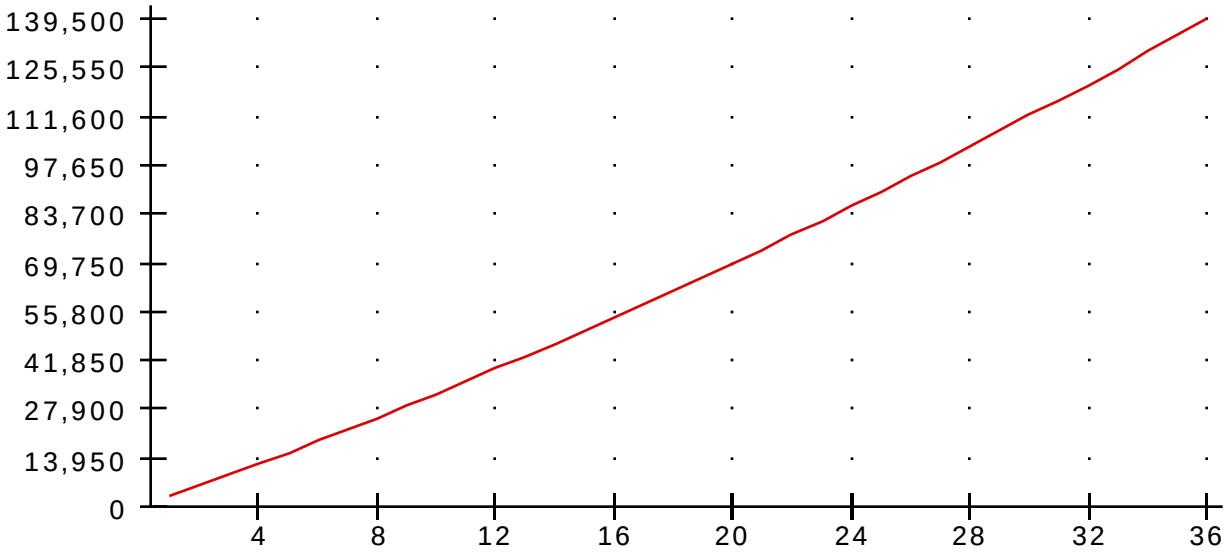
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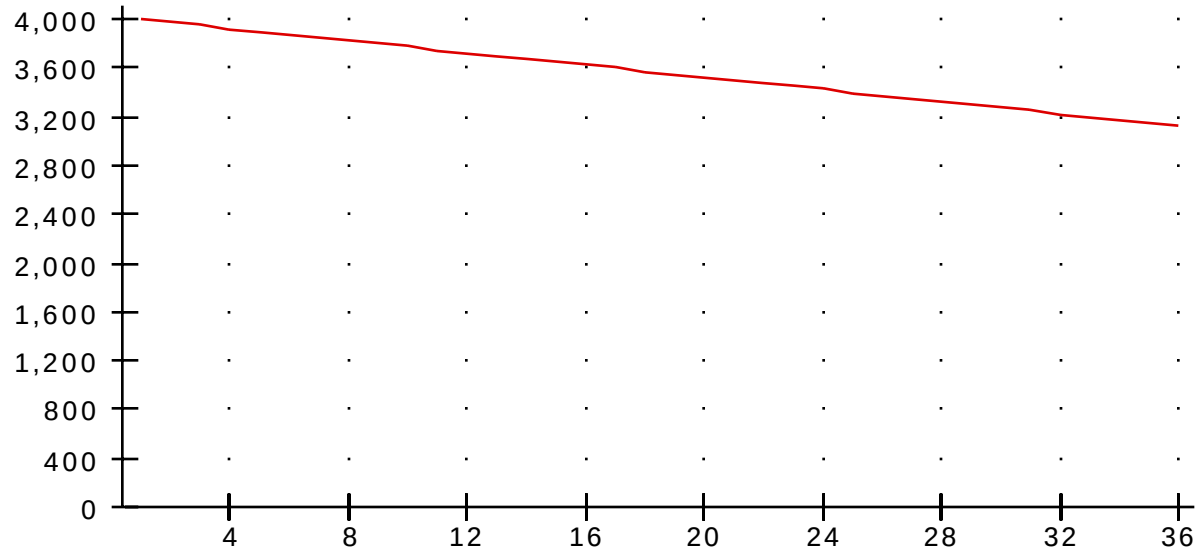
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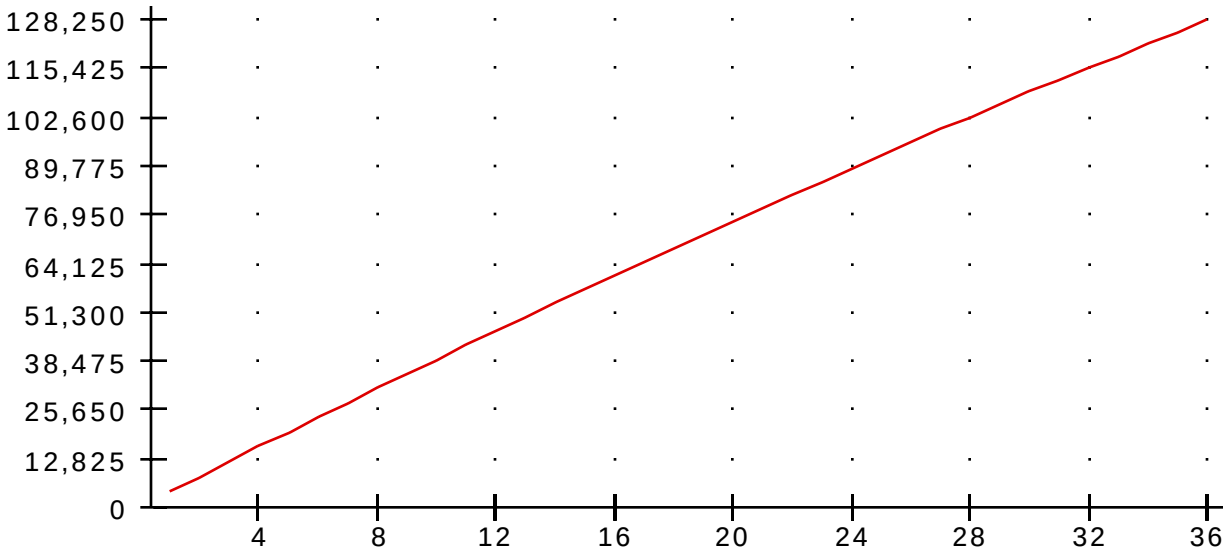
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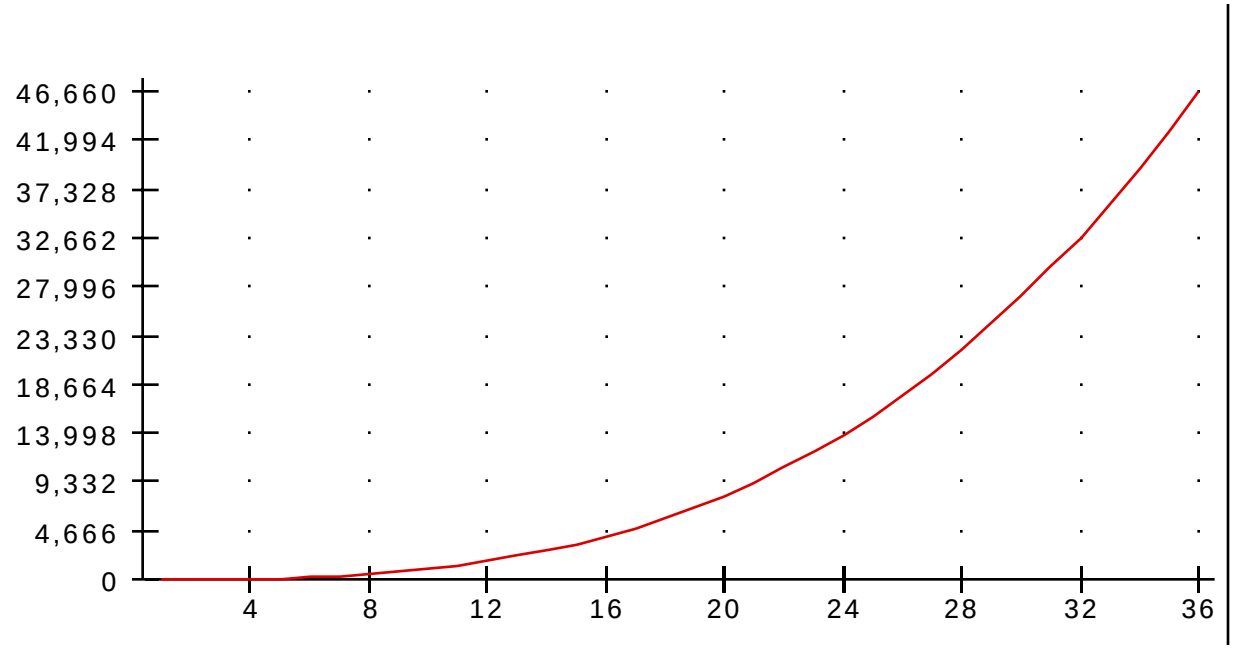
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37,328
32,662
27,996
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18,664
13,998
9,332
4,666
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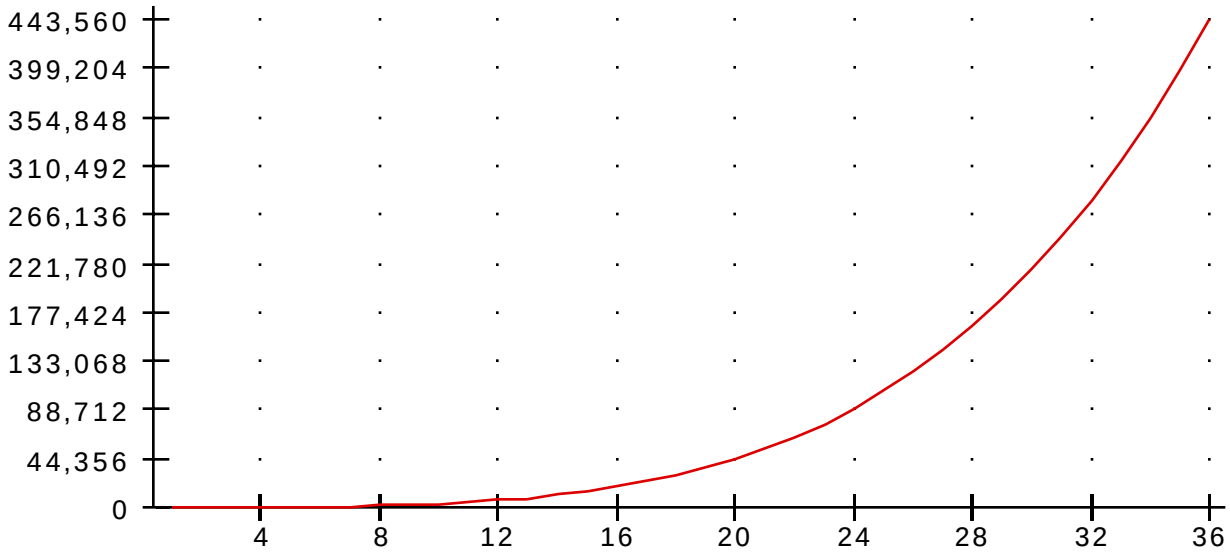
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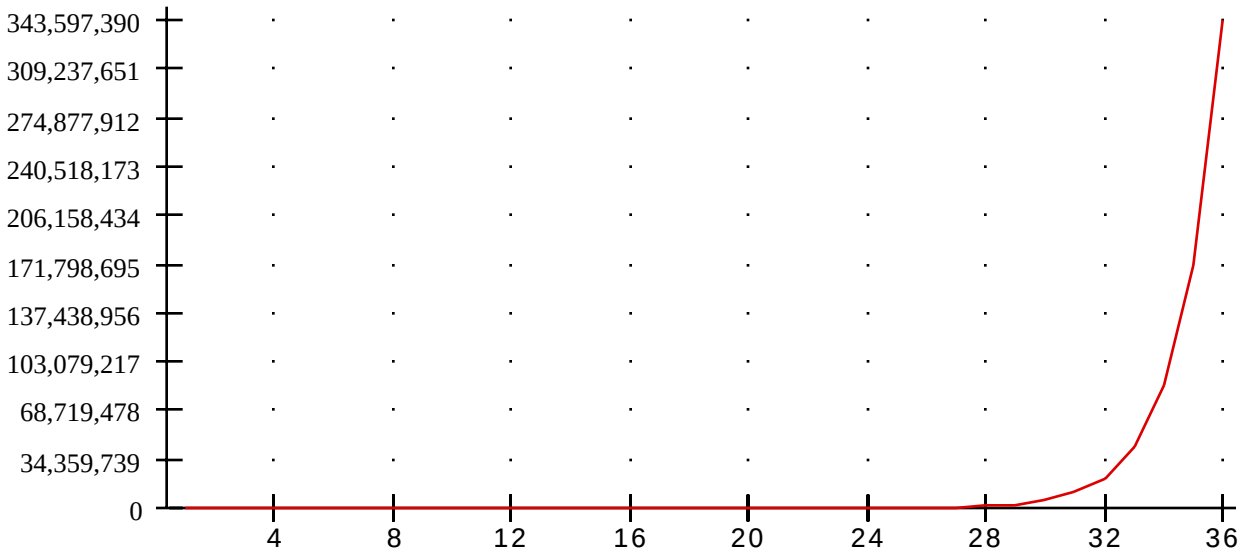
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240,518,173
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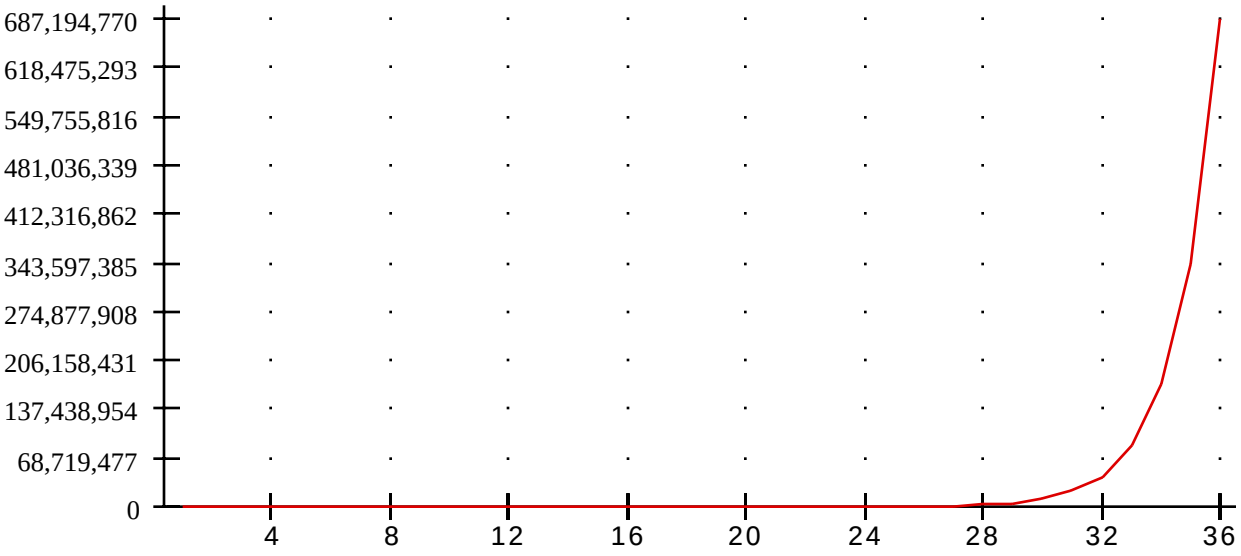
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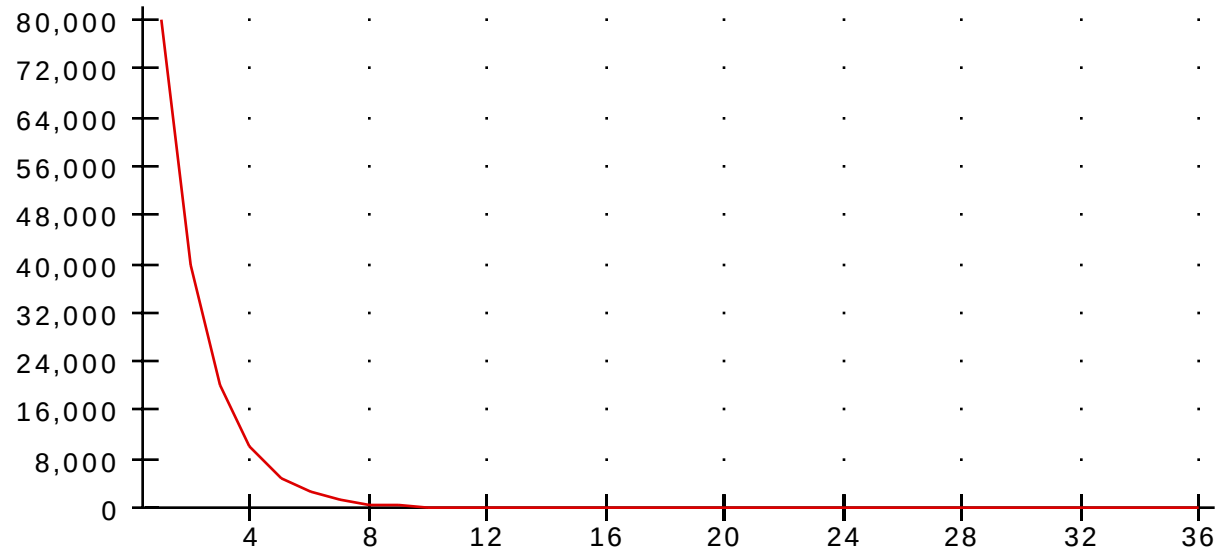
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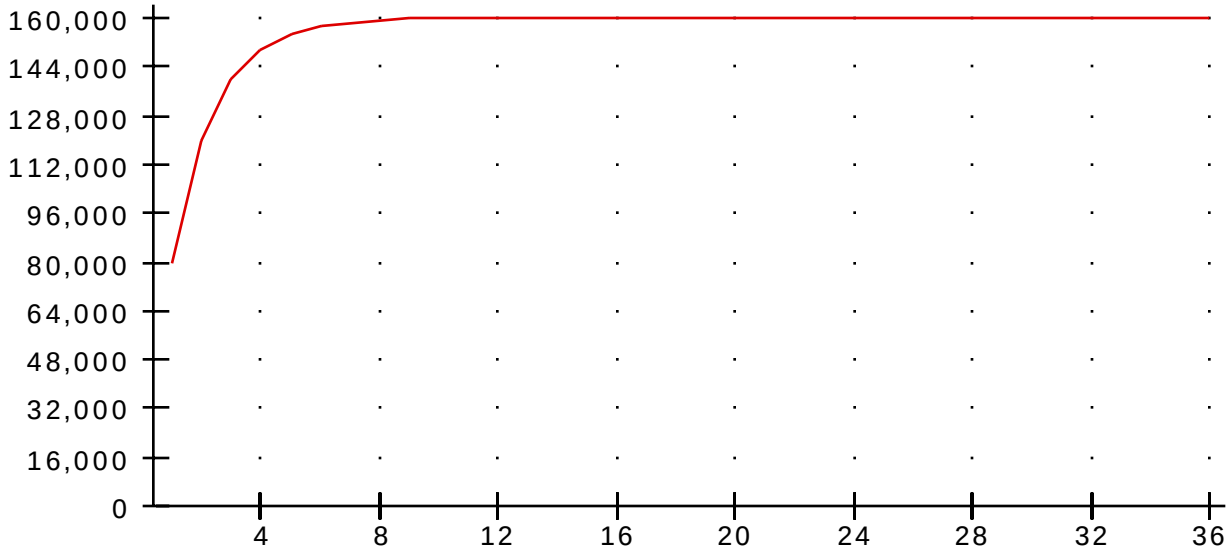
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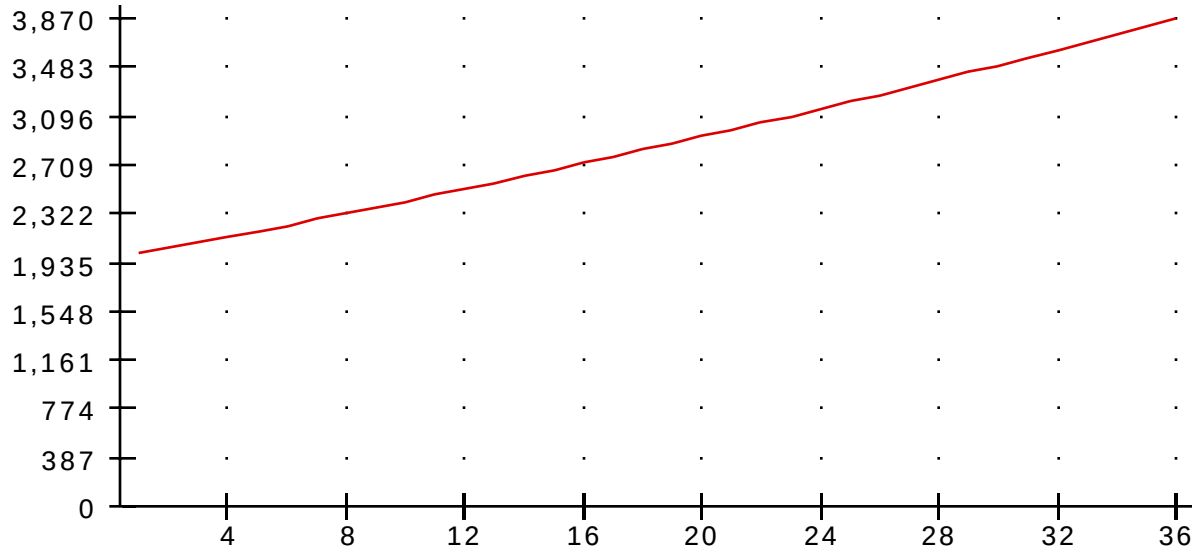
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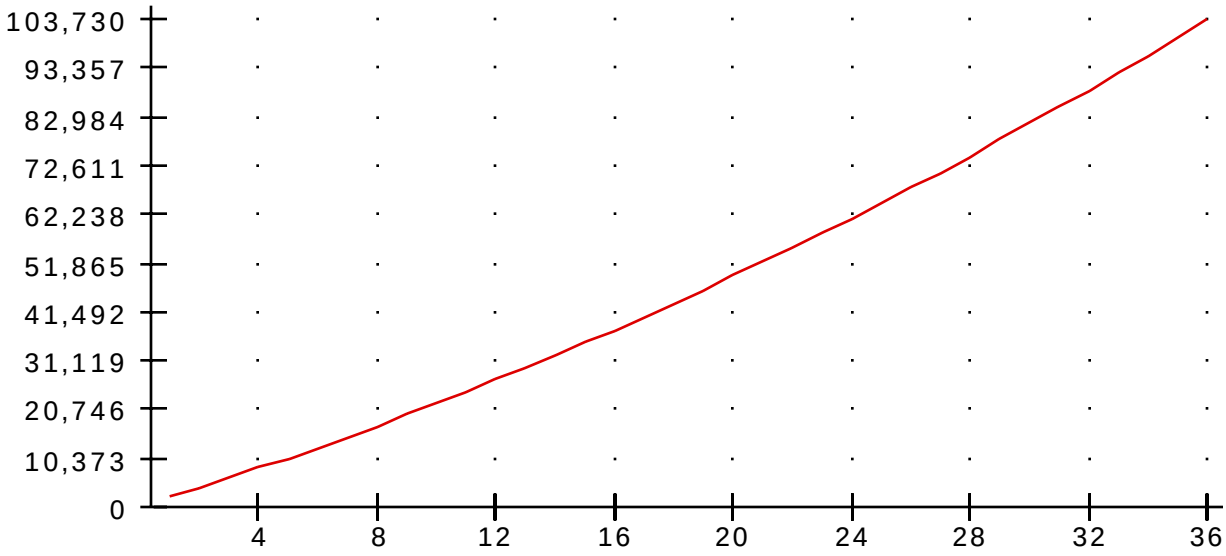
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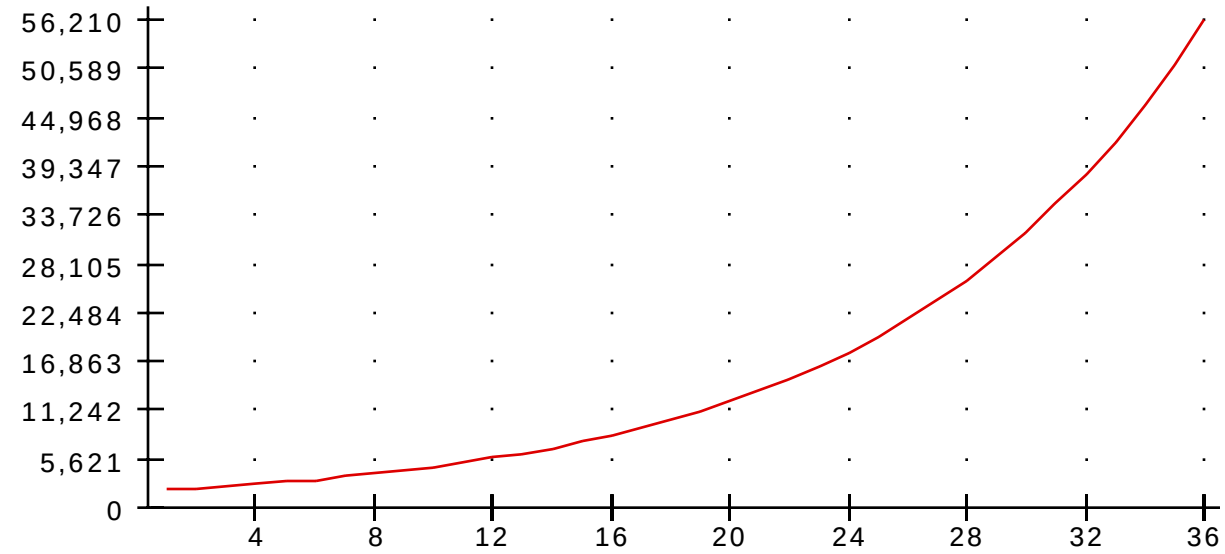
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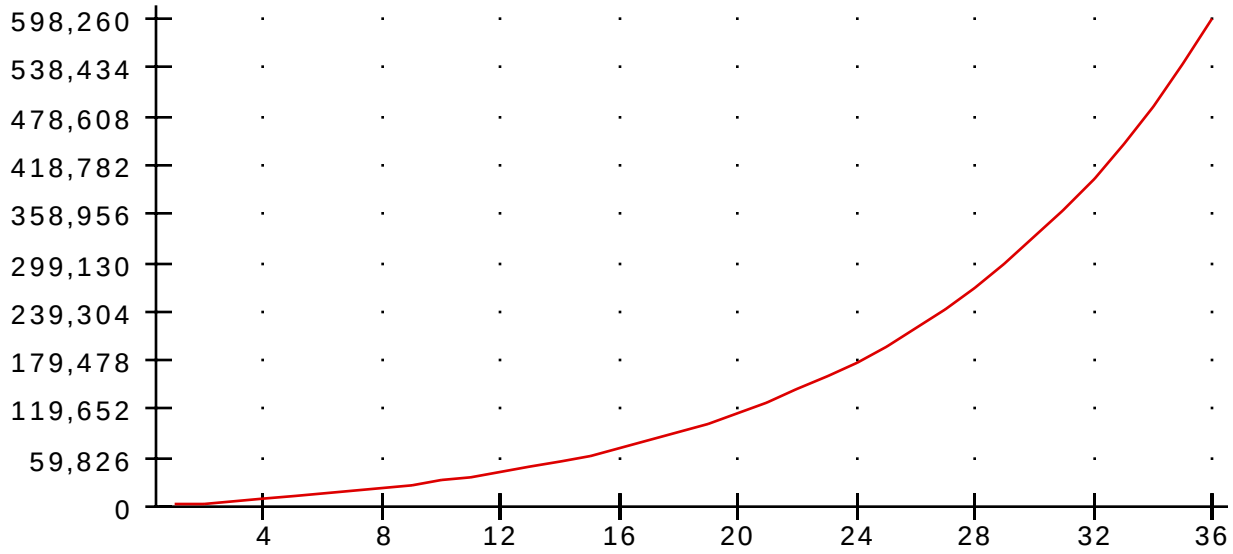
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418,782
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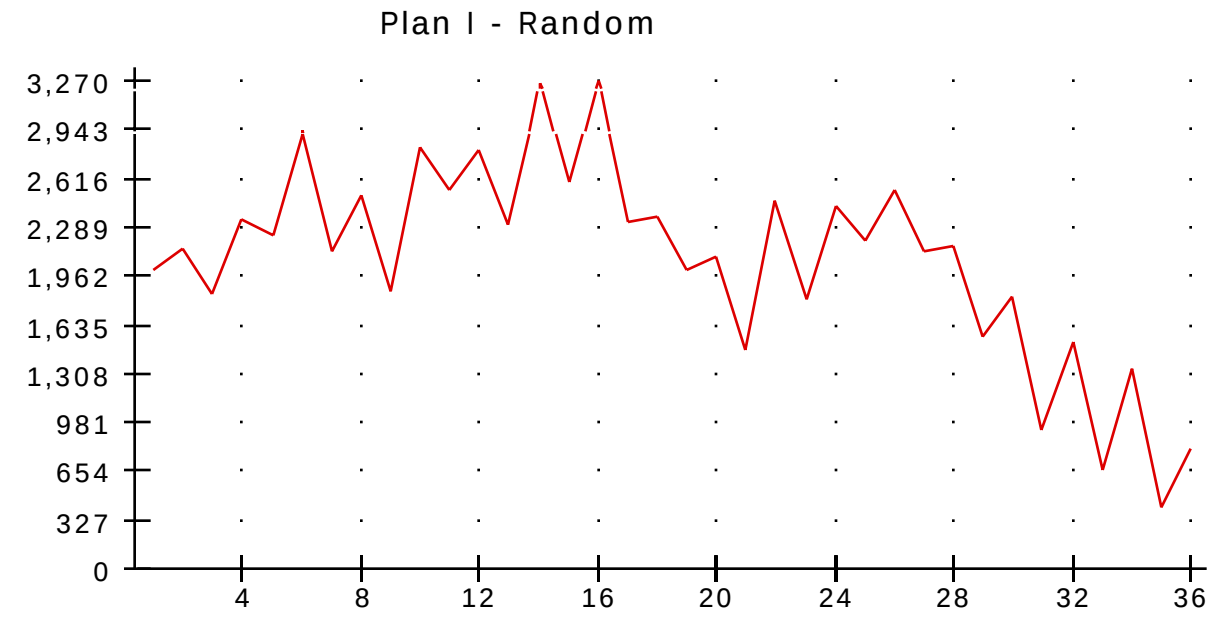
Plan I - Random
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2,616
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1,308
981
654
327
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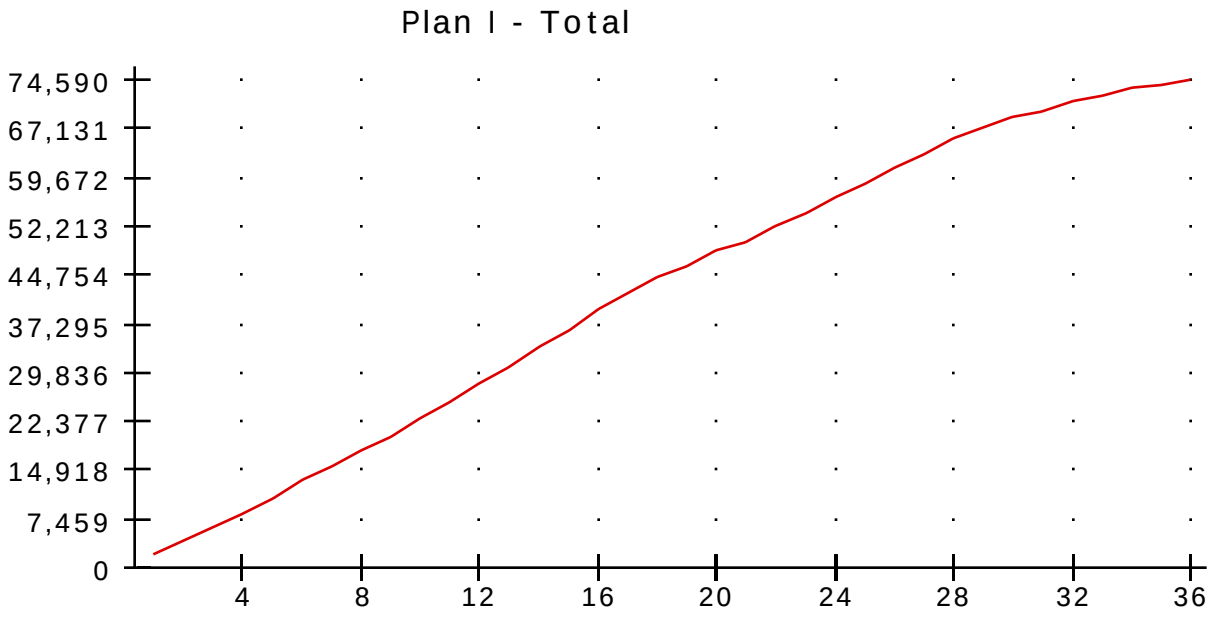
Plan I - Total
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52,213
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37,295
29,836
22,377
14,918
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4
8
12
16
20
24
28
32
36